New technology, Human Capital and Growth for Developing Countries

Cuong Le Van, Manh-Hung Nguyen, Thai Bao Luong

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Centre d' Economie de la Sorbonne, Université Paris-1, Pantheon-Sorbonne

106-112 Bd de l' Hôpital, 75647 Paris Cedex 13, France. *

December 7, 2006

Abstract

We consider a developing country with three sectors in economy: consumption goods, new technology, and education. Productivity of the consumption goods sector depends on new technology and skilled labor used for production of the new technology. We show that there might be three stages of economic growth. In the first stage the country concentrates on production of consumption goods; in the second stage it requires the country to import both physical capital to produce consumption goods and new technology capital to produce new technology; and finally the last stage is one where the country needs to import new technology capital and invest in the training and education of high skilled labor in the same time.

Keywords: Optimal growth model, New technology capital, Human Capital, Developing country.

JEL Classification: D51, E13

*The authors thank Tu-Anh Nguyen for thoughtful remarks

1 Introduction

Technology and adoption of technology have been important subjects of research in the literature of economic growth in recent years. Sources of technical progress might be domestic or/and international though there always exists believes amongst economic professionals that there is an important difference between developed and developing countries, i.e. the first one innovates and exports technology while the second one imports and copies. For developing countries, the adoption of technology from international market is vital since it might be the only way for them to improve their productivity growth and technical progress (Romer [1997, 1990]). But it is even more important to stress that these countries also need to care about their human capital (Lucas [1988]) which might be the key factor that determines whether a country, given their level of development, can take off or might fall into proverty trap.

This line of argument comes from the fact that the developing countries today are facing a dilemma of whether to invest in physical, technological, and human capital. As abundantly showed in literature (e.g. Barro [1997], Barro & Sala-i-Martin [1995], Eaton & Kortum [2000], Keller [2001], Kumar [2003], Kim & Lau [1994], Lau & Park [2003]) developing countries are not convergent in their growth paths and in order to move closer to the world income level, a country needs to have a certain level in capital accumulation.

In their recent work, Bruno et al. [2006] point out the conditions under which a developing country can optimally decide to either concentrate their whole resources on physical capital accumulation or spend a portion of their national wealth to import technological capital. These conditions are related to the nation's stage of development which consists of level of wealth and endowment of human capital and thresholds at which the nation migh switch to another stage of development. However, in their model, the role of education that contributes to accumulation of human capital and effecient use of technological capital is not fully explored.

In this paper we extend their model by introducing an educational sector with which the developing country would invest in to train more skilled labor. We show that the country once reaches a critical value of wealth will have to consider the import of new technology. At this point, the country can either go on with its existing production technology or improve it by purchasing new technology capital in order to produce new technology. But when the level of wealth passes this value it is always optimal for the country to use new technology which requires high skilled workers. We show further that with possibility of investment in human capital and given "good" conditions on the qualities of the new technology, production process, and/or the number of skilled workers there exists alternatives for the country either to purchase new technology and spend money in training high skilled labor or only purchase new technology but not to spend on formation of labor. Following this direction, we can determine the level of wealth at which the decision to invest in training and education has to be made. In this context, we can show that the critical value of wealth is inversely related to productivity of the new technology sector, number of skilled workers, and indicator of the impact of the new technology sector on the consumption goods sector but proportionally related to price of the new technology capital. In the whole, the paper allows us to determine the optimal share of the country's investment in physical capital, new technology capital and human capital formation in the long-run growth path. Two main results can be pointed out: (1) the richer a country is, the more money will be invested in new technology and training and education, (2) and more interestingly, the share of investment in human capital will increase with the wealth while the one for physical and new technology capitals will decrease. In any case, the economy will grow without bound.

The paper is organized as follows. Section 2 is for the presentation of one period model. Section 3 deals with dynamic properties with infinitely lived representative consumer.

2 The one period model

Consider an economy where exists three sectors: domestic sector which produces an aggregate good Y_d, new technology sector with output Y_e and education sector characterized by a function $h(T)$ where T is the expenditure on training and education. The output Y_e is used by domestic sector to increase its total productivity. The production functions of two sectors are Cobb-Douglas, i.e, $Y_d = \Phi(Y_e)K_d^{\alpha_d}L_d^{1-\alpha_d}$ and $Y_e = A_e K_e^{\alpha_e}L_e^{1-\alpha_e}$ where $\Phi(.)$ is a non decreasing function which satisfies $\Phi(0) = x_0 > 0$, K_d, K_e, L_d, L_e and A_e be the physical capital, the technological capital, the low-skilled labor, the high-skilled labor and the total productivity, respectively, $0 < \alpha_d < 1, 0 < \alpha_e < 1$.

We assume that this country imports capital good, the price of which is considered as numeraire. The price of the new technology sector is higher and equal to λ such that $\lambda \geq 1$. Assume that labor mobility between sectors is impossible and wages are exogenous.

Let S be available amount of money denoted to the capital goods purchase. We have:

$$K_d + \lambda K_e + p_T T = S.$$

For simplicity, we assume $p_T = 1$, or in other words T is measured in capital goods.

Thus, the budget constraint of the economy can be written as follows

$$K_d + \lambda K_e + T = S$$

where S be the value of wealth of the country in terms of consumption goods.

The social planner maximizes the following program

$$\max\ Y_d = \max\ \Phi(Y_e) K_d^{\alpha_d} L_d^{1-\alpha_d}$$

subject to

$$\begin{aligned} &Y_e = A_e K_e^{\alpha_e} L_e^{1-\alpha_e}, \\ &K_d + \lambda K_e + T = S, \\ &0 \leq L_e \leq L_e^* h(T), \\ &0 \leq L_d \leq L_d^*. \end{aligned}$$

where h is the education technology.

Assume that $h(.)$ is an increasing concave function and $h(0) = h_0 > 0$. Let

$$\Delta = \{(\theta, \mu) : \theta \in [0,1], \mu \in [0,1], \theta + \mu \leq 1\}.$$

¿From the budget constraint, we can define $(\theta, \mu) \in \Delta$:

$$\lambda K_e = \theta S\ , K_d = (1 - \theta - \mu) S \text{ and } \ T = \mu S.$$

Observe that since the objective function is strictly increasing, at the optimum, the constraints will be binding. Let $L_e = L_e^* h$, $L_d = L_d^*$, then we have the following problem

$$\max_{(\theta,\mu)\in\Delta} \Phi(r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e})(1-\theta-\mu)^{\alpha_d} S^{\alpha_d} L_d^{*1-\alpha_d}.$$

where $r_e = \frac{A_e}{\lambda^{\alpha_e}} L_e^{*1-\alpha_e}$.

Let

$$\psi(r_e, \theta, \mu, S) = \Phi(r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e})(1-\theta-\mu)^{\alpha_d} L_d^{*1-\alpha_d}.$$

The problem now is equivalent to

$$\max_{(\theta,\mu)\in\Delta} \psi(r_e, \theta, \mu, S). \tag{P}$$

Since the function ψ is continuous in θ and μ, there will exist optimal solutions. Denote

$$F(r_e, S) = \max_{(\theta,\mu)\in\Delta} \psi(r_e, \theta, \mu, S).$$

Then by Maximum Theorem, F is continuous and $F(r_e, S) \geq x_0 L_d^{*1-\alpha_d}$.

Suppose that function $\Phi(x)$ is a constant in an initial phase and increasing linear afterwards:

$$\Phi(x) = \begin{cases} x_0 \text{ if } x \leq X \\ x_0 + a(x - X) \text{ if } x \geq X, a > 0. \end{cases}$$

Define

$$B = \{S \geq 0 : F(r_e, S) = x_0 L_d^{*1-\alpha_d}\},$$

Lemma 1 *B is a nonempty compact set.*

Proof: B is a nonempty since $0 \in B$. Of course B is closed since the function F is continuous. Let us prove that B is bounded. If not, take a sequence S^n in B and converging to $+\infty$ when $n \to +\infty$. Fix some $(\theta_0, \mu_0) \in \Delta$. Since $\{S^n\}$ $\in B$, we have

$$\psi(r_e, \theta_0, \mu_0, S^n) \leq F(r_e, S^n) = x_0 L_d^{*1-\alpha_d}.$$

Let $n \to +\infty$ then

$$\psi(r_e, \theta_0, \mu_0, S^n) \to +\infty.$$

A contradiction. Therefore, B is bounded. ■

Remark 1 *Observe that $F(r_e, S) \geq x_0 L_d^{*1-\alpha_d}$. If the optimal value for θ equals 0 then the one for μ is also 0 and $F(r_e, S) = x_0 L_d^{*1-\alpha_d}$.*

The following proposition shows that if S is small, then the country will not invest in new technology and human capital. When S is large, then it will invest in new technology.

Proposition 1 *i) There exists $\underline{S} > 0$ such that if $S \leq \underline{S}$ then $\theta = 0$ and $\mu = 0$.*

ii) There exists $\overline{S}$ such that if $S > \overline{S}$ then $\theta > 0$.

Proof: For any S, denote by $\theta(S)$, $\mu(S)$ the corresponding optimal values for θ and μ.

(i) Let $\underline{S}$ satisfies

$$r_e \underline{S}^{\alpha_e} h(\underline{S})^{1-\alpha_e} = X,$$

Then for any $(\theta, \mu) \in \Delta$, for any $S \leq \underline{S}$,

$$r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e} \leq X$$

and $(\theta(S), \mu(S)) = (0, 0)$.

(ii) Fix $\mu = 0$ and $\theta \in (0, 1)$. Then $\psi(r_e, \theta, 0, S) \to +\infty$ when $S \to +\infty$. Let $\overline{S}$ satisfy $\psi(r_e, \theta, 0, \overline{S}) > x_0 L_d^{*1-\alpha_d}$. Obviously, $F(r_e, \overline{S}) \geq \psi(r_e, \theta, 0, \overline{S}) >$

$x_0 L_d^{*1-\alpha_d}$, and $\theta(\overline{S}) > 0$. If not, then $\mu(\overline{S}) = 0$ and $F(r_e, \overline{S}) = x_0 L_d^{*1-\alpha_d}$ (see Remark 1). ■

Now, let us define

$$S^c = \max\{S \geq 0 : S \in B\}.$$

It is obvious that $0 < S^c < +\infty$, since $S^c \geq \underline{S} > 0$ and B is compact.

Proposition 2 *If $S < S^c$ then $\theta(S) = 0$ and $\mu(S) = 0$, and if $S > S^c$ then $\theta(S) > 0$.*

Proof: Note that for any $S \geq 0$ we have

$$F(r_e, S) \geq x_0 L_d^{*1-\alpha_d}.$$

If $S < S^c$ then for any $(\theta, \mu) \in \Delta$,

$$\psi(r_e, \theta, \mu, S) \leq \psi(r_e, \theta, \mu, S^c)$$

which implies

$$F(r_e, S) \leq F(r_e, S^c) = x_0 L_d^{*1-\alpha_d}.$$

Thus,

$$F(r_e, S) = x_0 L_d^{*1-\alpha_d}.$$

Let $S_0 < S^c$. Assume there exists two optimal values for (θ, μ) which are $(0, 0)$ and (θ_0, μ_0) with $\theta_0 > 0$. We have $F(r_e, S_0) = x_0 L_d^{*1-\alpha_d} = \psi(r_e, \theta_0, \mu_0, S_0)$. We must have $r_e \theta_0^{\alpha_e} S_0^{\alpha_e} h(\mu_0 S_0)^{1-\alpha_e} > X$ (if not, $\Phi(r_e, \theta_0, \mu_0, S_0) = x_0$ and $\theta_0 = 0, \mu_0 = 0$.)

Since $\theta_0 > 0$, we have $r_e \theta_0^{\alpha_e} (S^c)^{\alpha_e} h(\mu_0 S_0)^{1-\alpha_e} > r_e \theta_0^{\alpha_e} S_0^{\alpha_e} h(\mu_0 S_0)^{1-\alpha_e} > X$. Hence

$$\begin{aligned} x_0 L_d^{*1-\alpha_d} = F(r_e, S^c) &\geq \psi(r_e, \theta_0, \mu_0, S^c) \\ &> \psi(r_e, \theta_0, \mu_0, S_0) = x_0 L_d^{*1-\alpha_d} \end{aligned}$$

which is a contradiction.

Therefore, if $S > S^c$ then

$$F(r_e, S) > x_0 L_d^{*1-\alpha_d}$$

which implies $\theta(S) > 0$. ■

The following proposition shows that, when the quality of the training technology (measured by the marginal productivity at the origin $h'(0)$) is very high then for $S > S^c$ the country will invest both in new technology and in human capital.

Proposition 3 *If $h'(0) = +\infty$, then for all $S > S^c$, we have $\theta(S) > 0, \mu(S) > 0$.*

Proof: Take $S > S^c$. From the previous proposition, $\theta(S) > 0$. Assume $\mu(S) = 0$. For short, denote $\theta^* = \theta(S)$. Define

$$F^0(r_e, S, \theta^*, 0) = \max_{0 \le \theta \le 1} \psi(r_e, \theta, 0, S) = \Phi(r_e \theta^{*\alpha_e} S^{\alpha_e} h(0)^{1-\alpha_e})(1-\theta^*)^{\alpha_d} L_d^{*1-\alpha_d}.$$

and consider a feasible couple (θ, μ) in Δ which satisfies $\theta^* = \theta + \mu$. Denote

$$F^1(r_e, S, \theta, \mu) = \Phi(r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e})(1-\theta^*)^{\alpha_d} L_d^{*1-\alpha_d}.$$

We then have

$$\begin{aligned}
&\frac{F^1(r_e, S, \theta, \mu) - F^0(r_e, S, \theta^*, 0)}{(1-\theta^*)^{\alpha_d} L_d^{*1-\alpha_d}} = \\
&\Phi(r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e}) - \Phi(r_e \theta^{*\alpha_e} S^{\alpha_e} h(0)^{1-\alpha_e}) \\
= \;& r_e S^{\alpha_e} [\theta^{\alpha_e} h(\mu S)^{1-\alpha_e} - \theta^{*\alpha_e} h(\mu S)^{1-\alpha_e} + \theta^{*\alpha_e} h(\mu S)^{1-\alpha_e} - \theta^{*\alpha_e} h(0)^{1-\alpha_e}].
\end{aligned}$$

By the concavity of $h(x)$ and $f(x) = x^{\alpha_e}$, we obtain

$$\begin{aligned}
&F^1(r_e, S, \theta, \mu) - F^0(r_e, S, \theta^*, 0) \ge \\
&r_e S^{\alpha_e} \mu h(\mu S)^{-\alpha_e} [-\alpha_e h(\mu S)(\theta^* - \mu)^{\alpha_e - 1} + S(1-\alpha_e)\theta^{*\alpha_e} h'(\mu S)].
\end{aligned}$$

Let $\mu \to 0$. We have $h'(\mu S) \to +\infty$. The expression in the brackets will converge to $+\infty$, and we get a contradiction with the optimality of θ^*. ■

When $h'(0)$ is finite, we are not ensured that the country will invest in human capital when $S > S^c$. But it will do if it is sufficiently rich.

Proposition 4 *Assume $h'(0) < +\infty$. Then there exists S^M such that $\mu(S) > 0, \theta(S) > 0$ for every $S > S^M$.*

Proof: Assume that $\mu(S) = 0$ for any $S \in \{S^1, S^2, ..., S^n, ...\}$ where the infinite sequence $\{S^n\}_n$ is increasing, converges to $+\infty$ and satisfies $S^1 > S^c$. For short, denote $\theta = \theta(S)$. Then we have the following FOC:

$$\frac{a r_e \theta^{\alpha_e - 1} S^{\alpha_e} h(0)^{1-\alpha_e} \alpha_e}{x_0 + a[r_e \theta^{\alpha_e} S^{\alpha_e} h(0)^{1-\alpha_e} - X]} = \frac{\alpha_d}{1-\theta}, \tag{1}$$

and

$$\frac{a r_e \theta^{\alpha_e} S^{\alpha_e + 1} h'(0) h(0)^{-\alpha_e}(1-\alpha_e)}{x_0 + a[r_e \theta^{\alpha_e} S^{\alpha_e} h(0)^{1-\alpha_e} - X]} \le \frac{\alpha_d}{1-\theta}. \tag{2}$$

Equation (1) implies

$$\frac{a r_e \theta^{\alpha_e - 1} h(0)^{1-\alpha_e} \alpha_e}{\frac{x_0}{S^{\alpha_e}} + a[r_e \theta^{\alpha_e} h(0)^{1-\alpha_e}]} \le \frac{\alpha_d}{1-\theta}. \tag{3}$$

If $\theta \to 0$ when $S \to +\infty$, then the LHS of inequality (3) converges to infinity while the RHS converges to α_d: a contradiction. Thus θ will be bounded away from 0 when S goes to infinity.

Combining equality (1) and inequality (2) we get

$$h'(0)(1-\alpha_e)S \leq h_0 \alpha_e \theta^{-1}. \tag{4}$$

When $S \to +\infty$, we have a contradiction since the LHS of (4) will go to infnity while the RHS will be bounded from above. That means there exists S_M such that for any $S \geq S_M$, we have $\mu(S) > 0$. ■

Remark 2 *Let $S > S^c$. For short, we denote μ and θ instead of $\mu(S)$ and $\theta(S)$. If $\mu > 0$ then we have the FOC:*

$$\frac{a r_e \theta^{\alpha_e - 1} S^{\alpha_e} h(\mu S)^{1-\alpha_e} \alpha_e}{x_0 + a[r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e} - X]} = \frac{\alpha_d}{1-\theta-\mu}, \tag{5}$$

and

$$\frac{a r_e \theta^{\alpha_e} S^{\alpha_e + 1} h'(\mu S) h(\mu S)^{-\alpha_e}(1-\alpha_e)}{x_0 + a[r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e} - X]} = \frac{\alpha_d}{1-\theta-\mu}. \tag{6}$$

The following proposition shows that, if $h'(0)$ is low, then the country will not invest in human capital when S belongs to some interval $(S^c,\ S^m)$.

Proposition 5 *There exists $\alpha > 0$ such that, if $h'(0) < \alpha$, then there exists $S^m > S^c$ such that $\mu(S) = 0, \theta(S) > 0$ for $S \in [S^c, S^m]$.*

Proof: Let θ^c and S^c satisfy the following equations

$$\frac{a r_e (\theta^c)^{\alpha_e - 1} (S^c)^{\alpha_e} h(0)^{1-\alpha_e} \alpha_e}{x_0 + a[r_e (\theta^c)^{\alpha_e} (S^c)^{\alpha_e} h(0)^{1-\alpha_e} - X]} = \frac{\alpha_d}{1-\theta^c}, \tag{7}$$

and

$$(x_0 + a[r_e (\theta^c)^{\alpha_e} (S^c)^{\alpha_e} h(0)^{1-\alpha_e} - X])(1-\theta^c)^{\alpha_d} = x_0. \tag{8}$$

Equality (7) is the FOC with respect to θ, while equality (8) states that $\psi(r_e, \theta^c, 0, S^c) = x_0 L_d^{*1-\alpha_d}$. Tedious computations show that equations (7) and (8) are equivalent to

$$a\ r_e (S^c)^{\alpha_e} h(0)^{1-\alpha_e} \alpha_e = \frac{\alpha_d x_0}{(\theta^c)^{\alpha_e - 1}(1-\theta^c)^{1+\alpha_d}} \tag{9}$$

and

$$x_0(1 + \frac{\alpha_d}{\alpha_e})\theta^c = x_0 - (x_0 - a\ X)(1-\theta^c)^{1-\alpha_d} \tag{10}$$

The LHS of equation (10) is linear, increases from 0 when $\theta^c = 0$ to $x_0(1 + \frac{\alpha_d}{\alpha_e})$ when $\theta^c = 1$. The RHS of this equation is a non-increasing function, when

$x_0 \le aX$, from aX when $\theta^c = 0$ to x_0 when $\theta^c = 1$. In this case, there exists a unique solution for θ^c in $(0,\ 1)$. When $x_0 > aX$, the RHS is a convex function which increases from aX when $\theta^c = 0$ to x_0 when $\theta^c = 1$. Again, there exists a unique solution for θ^c in $(0,\ 1)$. Once θ^c is determined, S^c is given by equation (9).

Now, if $h'(0) < \alpha = h(0)\frac{1}{\theta^c S^c}\frac{\alpha_e}{1-\alpha_e}$, then we get

$$\frac{ar_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e+1}h'(0)h(0)^{-\alpha_e}(1-\alpha_e)}{x_0 + a[r_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e}h(0)^{1-\alpha_e} - X]} < \frac{\alpha_d}{1-\theta^c}. \tag{11}$$

Relations (7), (8) and (11) give the the values of S^c and $\theta(S^c) = \theta^c$ and $\mu(S^c) = \mu^c = 0$. When $S > S^c$ and close to S^c, equality (7) and inequality (11) still hold. That means $\mu(S) = 0$ for any S close to S^c. ∎

Remark 3 *Let τ be defined by $T = \tau\lambda$, i.e. τ is the measure of T using the new technology capital as numeraire. In this case, condition $h'(0) < h(0)\frac{1}{\theta^c S^c}\frac{\alpha_e}{1-\alpha_e}$ is equivalent to $\frac{\partial Y_e}{\partial K_e} > (\frac{\partial Y_e}{\partial L_e})(\frac{\partial L_e}{\partial \tau})_{T=0}$.*

Proposition 6 *Assume $h'(0) < +\infty$. Let $S^1 > S^c$. If $\mu(S^1) = 0$, then for $S^2 < S^1$, we also have $\mu(S^2) = 0$.*

Proof: If $S^2 \le S^c$ then $\mu(S^2) = 0$ since $\theta(S^2) = 0$ (see Proposition 2). For short, we write $\theta_1 = \theta(S^1),\ \theta_2 = \theta(S^2),\ \mu_1 = \mu(S^1),\ \mu_2 = \mu(S^2)$.

Observe that (θ_1, S^1) satisfy (1) and (2), or equivalently (1) and (4). Equality (1) can be written as

$$h_0^{1-\alpha_e} ar_e[\alpha_e\theta_1^{\alpha_e-1} - (\alpha_e+\alpha_d)\theta_1^{\alpha_e}] = \frac{\alpha_d(x_0 - aX)}{S^{1\alpha_e}}. \tag{12}$$

If $x_0 - aX = 0$, then $\theta_1 = \frac{\alpha_e}{\alpha_e+\alpha_d}$. Take $\theta_2 = \theta_1$. If $S^2 < S^1$ then (θ_2, S^2) satisfy (1) and (4). That means they satisfy the FOC with $\mu_2 = 0$.

Observe that the LHS of equation (12) is a decreasing function in θ_1. Hence θ_1 is uniquely determined.

When $x_0 > aX$, if (θ_2, S^2) satisfy (12), with $S^2 < S^1$, then $\theta_2 < \theta_1$. In this case, (θ_2, S^2) also satisfy (4), and we have $\mu_2 = 0$.

When $x_0 < aX$, write equation (12) as:

$$h_0^{1-\alpha_e} ar_e[\alpha_e\theta_1^{-1} - (\alpha_e+\alpha_d)] = \frac{\alpha_d(x_0 - aX)}{(\theta_1 S^1)^{\alpha_e}}. \tag{13}$$

If (θ_2, S^2) satisfy (12), with $S^2 < S^1$, then $\theta_2 > \theta_1$. Since $x_0 < aX$, from (13), we have $\theta_2 S^2 < \theta_1 S^1$. Again (θ_2, S^2) satisfy (12) and (4). That implies $\mu_2 = 0$. ∎

From Proposition 6, we have as corollary, the following proposition.

Proposition 7 *Assume $h'(0) < +\infty$. Then there exists $\widehat{S} \geq S^c$ such that:*
(i) $S \leq \widehat{S} \Rightarrow \mu(S) = 0$,
(ii) $S > \widehat{S} \Rightarrow \mu(S) > 0$.

Proof: Let

$$\widetilde{S} = \max\{S_m : S_m > S^c, \text{ and } S \leq S_m \Rightarrow \mu(S) = 0\},$$

and

$$\widetilde{\widetilde{S}} = \inf\{S_M : S_M > S^c, \text{ and } S > S_M \Rightarrow \mu(S) > 0\}.$$

From Propositions 4 and 5, the sets $\{S_m : S_m > S^c, \text{ and } S \leq S_m \Rightarrow \mu(S) = 0\}$ and $\{S_M : S_M > S^c, \text{ and } S > S_M \Rightarrow \mu(S) > 0\}$ are not empty. From Proposition 6, we have $\widetilde{\widetilde{S}} \geq \widetilde{S}$. If $\widetilde{\widetilde{S}} > \widetilde{S}$, then take $S \in (\widetilde{S}, \widetilde{\widetilde{S}})$. From the definitions of $\widetilde{S}$ and $\widetilde{\widetilde{S}}$, there exist $S_1 < S$, $S_2 > S$ such that $\mu(S_1) > 0$ and $\mu(S_2) = 0$. But that contradicts Proposition 6. Hence $\widetilde{\widetilde{S}} = \widetilde{S}$. Put $\widehat{S} = \widetilde{\widetilde{S}} = \widetilde{S}$ and conclude. ■

Let us recall $r_e = \frac{A_e L_e^{*(1-\alpha_e)}}{\lambda^{\alpha_e}}$ where A_e is the productivity of the new technology sector, λ is the price of the new technology capital and L_e^* is the number of skilled workers.

Recall also the productivity function of the consumption goods sector $\Phi(x) = x_0 + a(x - X)$ if $x \geq X$. The parameter $a > 0$ is an indicator of the impact of the new technology product x on the this productivity. We will show in the following proposition that the critical value S^c diminishes when r_e increases, i.e. when the productivity A_e and/or the number of skilled workers increase, and /or the price of the new technology capital λ decreases, and /or the impact indicator a increases.

Proposition 8 *Assume $h(z) = h_0 + bz$, with $b > 0$. Let $\theta^c = \theta(S^c)$, $\mu^c = \mu(S^c)$. Then*
(i) $\mu^c = 0$, θ^c does not depend on a and r_e.
(ii) S^c decreases if a or/and r_e increases.

Proof: From Proposition 7, we have $\mu^c = 0$. In this case, θ^c and S^c satisfy equation (5) and, since $S^c \in B$, we also have $F(r_e, S^c) = \psi(r_e, \theta^c, 0, S^c) = x_0 L_d^{*1-\alpha_d}$.

Explicitly, we have

$$\frac{a r_e (\theta^c)^{\alpha_e - 1} (S^c)^{\alpha_e} h_0^{1-\alpha_e} \alpha_e}{x_0 + a[r_e (\theta^c)^{\alpha_e} (S^c)^{\alpha_e} h_0^{1-\alpha_e} - X]} = \frac{\alpha_d}{1 - \theta^c}$$

and

$$(x_0 + a[r_e (\theta^c)^{\alpha_e} (S^c)^{\alpha_e} h_0^{1-\alpha_e} - X])(1 - \theta^c)^{\alpha_d} = x_0 \quad (14)$$

Tedious computations show that θ^c satisfies the equation

$$\alpha_e[1 - \frac{x_0 - aX}{x_0}(1-\theta)^{\alpha_d+1}] = \theta(\alpha_d + \alpha_e)$$

If $x_0 > aX$, then the LHS is a strictly concave function which increases from $\frac{\alpha_e aX}{x_0}$ when $\theta = 0$ to α_e when $\theta = 1$. The RHS is linear increasing, equal to 0 at the origin and to $\alpha_d + \alpha_e$ when $\theta = 1$. Therefore, there exists a unique solution $\theta^c \in (0, 1)$.
If $x_0 < aX$, then the LHS is a strictly convex function which decreases from $\frac{\alpha_e aX}{x_0}$ when $\theta = 0$ to α_e when $\theta = 1$. The RHS is linear increasing, equal to 0 at the origin and to $\alpha_d + \alpha_e$ when $\theta = 1$. Therefore, there exists a unique solution $\theta^c \in (0, 1)$.
If $x_0 = aX$, then $\theta^c = \frac{\alpha_e}{\alpha_e+\alpha_d}$.
In any case, θ^c does not depend on a and r_e.

Equation (14) gives:

$$ar_e(S^c)^{\alpha_e} = [x_0(\frac{1}{(1-\theta^c)^{\alpha_d}} - 1) + aX]\frac{1}{(\theta^c)^{\alpha_e} h_0^{1-\alpha_e}} \tag{15}$$

We see immediatly that S^c is a decreasing function in a and r_e. ■

The following proposition shows that the optimal shares θ, μ converge when S goes to infinity and the ratio between physical capital and the total of new technology capital and the amount devoted to human capital formation decreases when S increases.

Proposition 9 *Assume $h(z) = h_0 + bz$, with $b > 0$. Then the optimal shares $\theta(S), \mu(S)$ converge to $\theta_\infty, \mu_\infty$ when S converges to $+\infty$. Consider $\widehat{S}$ in Proposition 7. Then*

(i) If $x_0 < aX$, $\theta(S)$ decreases from θ^c to $\widehat{\theta} = \theta(\widehat{S})$ when S goes from S^c to $\widehat{S}$. The sum $\theta(S) + \mu(S)$ decreases when S increases from S^c to $\widehat{S}$.

(ii) If $x_0 \geq aX$, $\theta(S)$ increases from θ^c to $\widehat{\theta} = \theta(\widehat{S})$ when S goes from S^c to $\widehat{S}$. The sum $\theta(S) + \mu(S)$ increases when S increases from S^c to $\widehat{S}$.

(iii)If ar_e is large enough, $\theta(S)$ decreases from $\widehat{\theta}$ to $\theta_\infty = \frac{\alpha_e}{1+\alpha_d}$ when S increases from $\widehat{S}$ to $+\infty$. The sum $\theta(S) + \mu(S)$ increases with S for $S > \widehat{S}$. Moreover, $\mu(S)$ also increases with S for $S > \widehat{S}$.

Proof: For short, write θ, μ instead of $\theta(S), \mu(S)$. Consider $\widehat{S}$ in Proposition 7. When $S \leq S^c$, we have $\theta = 0, \mu = 0$. When $S^c < S \leq \widehat{S}$, then $\mu = 0$ (Proposition 7). But θ satisfies equation (1) which can be written as

$$ar_e\theta^{\alpha_e - 1}h(0)^{1-\alpha_e}\alpha_e = \frac{\alpha_d(x_0 - aX)}{S^{\alpha_e}} + (\alpha_d + \alpha_e)ar_e\theta^{\alpha_e}h(0)^{1-\alpha_e}.$$

The LHS is a decreasing function while the RHS is increasing. It is easy to check that the unique solution exists and is in $(0,1)$. Differentiate with respect to S to see that θ increases with S if $x_0 > aX$ and decreases with S if $x_0 < aX$. When $x_0 = aX$, we obtain $\theta = \frac{\alpha_e}{\alpha_e+\alpha_d}$. Thus the sum $\theta + \mu$ increases with S when $S \in (S^c,\ \widehat{S})$ if $x_0 > aX$ and decreases if $x_0 < aX$.
Now consider the case where $S > \widehat{S}$. Then (θ, μ) satisfy equations (5) and (6) which can be written as follows:

$$\theta(\alpha_d + \alpha_e) = -\alpha_e\mu + [\alpha_e - \frac{\alpha_d(x_0 - aX)\alpha_e{}^{-\alpha_e}}{ar_e S(1-\alpha_e)^{1-\alpha_e} b^{1-\alpha_e}}] \tag{16}$$

and

$$\theta(1 - \alpha_e) = \alpha_e\mu + \frac{\alpha_e h_0}{bS} \tag{17}$$

We obtain

$$\theta(1 + \alpha_d) = [\alpha_e - \frac{\alpha_d(x_0 - aX)\alpha_e{}^{-\alpha_e}}{ar_e S(1-\alpha_e)^{1-\alpha_e} b^{1-\alpha_e}} + \frac{h_0\alpha_e}{bS}] \tag{18}$$

and

$$\mu = \theta(\frac{1}{\alpha_e} - 1) - \frac{h_0}{bS}$$

Thus

$$\theta + \mu = \frac{1}{1+\alpha_d}[1 - \frac{\alpha_d}{\alpha_e}\frac{(x_0 - aX)\alpha_e{}^{-\alpha_e}}{ar_e S(1-\alpha_e)^{1-\alpha_e} b^{1-\alpha_e}}] - \frac{\alpha_d}{1+\alpha_d}\frac{h_0}{bS}.$$

If $x_0 \geq aX$, then $\theta + \mu$ increases with S. If $x_0 < aX$, then when ar_e is large enough, then $\theta + \mu$ is an increasing function in S.
It is obvious to see that θ decreases with S (equation (18)) if $x_0 < aX$, or if ar_e is large, when $x_0 > aX$.
When S converges to $+\infty$, then θ converges to $\theta_\infty = \frac{\alpha_e}{1+\alpha_d}$ and μ converges to $\mu_\infty = \frac{1-\alpha_e}{1+\alpha_d}$. ■

3 The Dynamic Model

In this section, we consider an economy with one infinitely lived representative consumer who has an intertemporal utility function with discount factor $\beta < 1$. At each period, her savings will be used to import physical capital or/and new technology capital and/or to invest in human capital. We suppose the capital depreciation rate equals 1.

The social planner will solve the following dynamic growth model

$$
\begin{aligned}
\max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\
\text{s.t} \quad & c_t + S_{t+1} \le \Phi(Y_{e,t}) K_{d,t}^{\alpha_d} L_{d,t}^{1-\alpha_d} \\
& Y_{e,t} = A_e K_{e,t}^{\alpha_e} L_{e,t}^{1-\alpha_e} \\
& K_{d,t} + \lambda K_{e,t} + T_t = S_t, \\
& 0 \le L_{e,t} \le L_e^* h(T_t), 0 \le L_{d,t} \le L_d^*. \\
& \text{ the initial resource } S_0 \text{ is given.}
\end{aligned}
$$

The problem is equivalent to

$$
\begin{aligned}
\max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\
\text{s.t} \quad & c_t + S_{t+1} \le H(r_e, S_t), \forall t,
\end{aligned}
$$

with

$$
H(r_e, S) = F(r_e, S) S^{\alpha_d}.
$$

where $r_e = \frac{A_e}{\lambda^{\alpha_e}} L_{e,t}^{*1-\alpha_e}$. Obviously, $H(r_e, .)$ is continuous, strictly increasing and $H(r_e, 0) = 0$.

As in the previous section, we shall use S^c defined as follows:

$$
S^c = \max\{S \ge 0 : F(r_e, S) = x_0 L_d^{*1-\alpha_d}\}
$$

where

$$
F(r_e, S_t) = \max_{0 \le \theta_t \le 1, 0 \le \mu_t \le 1} \psi(r_e, \theta_t, \mu_t, S_t).
$$

We shall make standard assumptions on the function u under consideration.

H2. The utility function u is strictly concave, strictly increasing and satisfies the Inada condition: $u^{'}(0) = +\infty, u(0) = 0$.

At the optimum, the constraints will be binding, the initial program is equivalent to the following problem

$$
\begin{aligned}
& \max \sum_{t=0}^{\infty} \beta^t u(H(r_e, S_t) - S_{t+1}) \\
\text{s.t} \quad & 0 \le S_{t+1} \le H(r_e, S_t), \forall t. \\
& S_0 > 0 \text{ given.}
\end{aligned}
$$

By the same arguments as in Bruno et al. [2005], we have the following property

Proposition 10 *i) Every optimal path is monotonic*

ii) Every optimal trajectory (S_t^*) *from* S_0 *can not converge to* 0.

Let denote θ_t^*, μ_t^* be the optimal capital shares among technological capital stock and expenditure on training,

$$\lambda K_{e,t}^* = \theta_t{}^* S_t^* \text{ and } \ T_t^* = \mu_t^* S_t^*.$$

We then obtain the main result of this paper:

Proposition 11 *Assume $h(z) = h_0 + bz$, with $b > 0$ and $\alpha_e + \alpha_d \geq 1$. If a or/and r_e are large enough then the optimal path $\{S_t^*\}_{t=1,+\infty}$ converges to $+\infty$ when t goes to infinity. Hence:*

(i) there exists T_1 such that

$$\theta_t^* > 0 \ \forall t \geq T_1$$

(ii) there exists $T_2 \geq T_1$ such that

$$\theta_t^* > 0 \ , \mu_t^* > 0, \ \forall t \geq T_2$$

The sum $\theta_t^* + \mu_t^*$ and the share μ_t^* increase when t goes to infinity and converge to values less than 1.

Proof: Let S^s be defined by

$$\alpha_d (S^s)^{\alpha_d - 1} x_0 L_d^{*1-\alpha_d} = \frac{1}{\beta}. \tag{1}$$

If $S_0 > \widehat{S}$ ($\widehat{S}$ is defined in Proposition 7) then $\theta_t^* > 0, \ \mu_t^* > 0$ for every t.
If $S_0 > S^c$ then $\theta_t^* > 0$ for every t. If S_t^* converges to infinity, then there exists T_2 where $S_{T_2}^* > \widehat{S}$ and $\theta_t^* > 0, \ \mu_t^* > 0$ for every $t \geq T_2$.
Now consider the case where $0 < S_0 < S^c$. Obviously, $\theta_0^* = 0$. It is easy to see that if a or/and r_e are large then $S^c < S^s$. If for any t, we have $\theta_t^* = 0$, we also have $K_{e,t}^* = 0 \ \forall t$, and the optimal path (S_t^*) will converge to S^s (see Le Van and Dana [2003]). But, we have $S^c < S^s$. Hence the optimal path $\{S_t^*\}$ will be non decreasing and will pass over S^c after some date T_1 and hence $\theta_t^* > 0$ when $t \geq T_1$.
If the optimal path $\{S_t^*\}$ converges to infinity, then after some date T_2, $S_t^* > \widehat{S}$ for any $t > T_2$ and $\theta_t^* > 0, \mu_t^* > 0$.

It remains to prove that the optimal path converges to infinity if a or/and r_e are large enough.

Since the utility function u satisfies the Inada condition $u'(0) = +\infty$, we have Euler equation:

$$u^{'}(c_t^*) = \beta u^{'}(c_{t+1}^*) H_s^{'}(r_e, S_{t+1}^*).$$

If $S_t^* \to \overline{S} < \infty$, then $c_t^* \to \overline{c} > 0$. From Euler equation, we get

$$H_s^{'}(r_e, \overline{S}) = \frac{1}{\beta}.$$

We will show that $H_s'(r_e, S) > \frac{1}{\beta}$ for nay $S > S^c$. We have

$$\begin{aligned} H_s'(r_e, S) &= F_s'(r_e, S)S^{\alpha_d} + \alpha_d F(r_e, S)S^{\alpha_d - 1} \\ &\geq F_s'(r_e, S)S^{\alpha_d}. \end{aligned}$$

From the envelope theorem we get:
$F_s'(r_e, S)S^{\alpha_d} =$

$$\begin{aligned} &[ar_e\theta^{*\alpha_e}(h^*(\mu S))^{-\alpha_e}(\alpha_e h(\mu^* S) + (1-\alpha_e)\mu^* S h'(\mu^* S))S^{\alpha_d+\alpha_e-1}] \\ &\times L_d^{*1-\alpha_d}(1-\theta^*-\mu^*)^{\alpha_d} \end{aligned}$$

When ar_e is large, from Proposition 9, we have $\theta^* \geq \underline{\theta} = \min\{\theta^c, \theta_\infty\}$ and $\theta^* + \mu^* \leq \overline{\zeta} = \max\{\theta^c, \theta_\infty + \mu_\infty\}$. We then obtain

$$\begin{aligned} H_s'(r_e, S) &\geq L_d^{*1-\alpha_d}(1-\theta^*-\mu^*)^{\alpha_d}[ar_e\theta^{*\alpha_e}(h^*(\mu S))^{1-\alpha_e}\alpha_e S^{\alpha_d+\alpha_e-1}] \\ &\geq L_d^{*1-\alpha_d}(1-\overline{\zeta})^{\alpha_d}[ar_e\underline{\theta}^{\alpha_e}(h^*(0))^{1-\alpha_e}\alpha_e(S^c)^{\alpha_d+\alpha_e-1}] \end{aligned}$$

since $h(x) \geq h(0)$ and $\alpha_d + \alpha_e - 1 \geq 0$.

If $\alpha_d + \alpha_e = 1$, then

$$H_s'(r_e, S) \geq L_d^{*1-\alpha_d}(1-\overline{\zeta})^{\alpha_d}[ar_e\underline{\theta}^{\alpha_e}(h^*(0))^{1-\alpha_e}\alpha_e], \tag{19}$$

and when ar_e becomes very large, the RHS of inequality (19) will be larger than $\frac{1}{\beta}$.

Now assume $\alpha_d+\alpha_e > 1$. From equation (15), the quantity $ar_e(S^c)^{\alpha_e}$ equals

$$\gamma = [x_0(\frac{1}{(1-\theta^c)^{\alpha_d}} - 1) + ax]\frac{1}{(\theta^c)^{\alpha_e}h_0^{1-\alpha_e}}$$

and

$$S^c = (\frac{\gamma}{ar_e})^{\frac{1}{\alpha_e}}.$$

We now have

$$H_s'(r_e, S) \geq L_d^{*1-\alpha_d}(1-\overline{\zeta})^{\alpha_d}\underline{\theta}^{\alpha_e}(h^*(0))^{1-\alpha_e}\alpha_e\gamma(\frac{\gamma}{ar_e})^{\frac{\alpha_d-1}{\alpha_e}}$$

It is obvious that, since $\alpha_d - 1 < 0$, when ar_e is large, we have $H_s'(r_e, S) > \frac{1}{\beta}$.

Remark 4 *To summarize, at low level of economic growth this country would only invest in physical capital but when the economy grows this country would need to invest not only in physical capital but also in first, new technology and then, formation of high skilled labor. Under some mild conditions on the quality of the new technology production process and on the supply of skilled workers, the optimal path (S_t^*) converges to $+\infty$, i.e. the country grows without bound. In this case, the share of investment in new technology and human capital $(\theta_t^* + \mu_t^*)$*

will increase while the one in physical capital will decrease (this is in accordance with the empirical results in Lau and Park (2003)). More interestingly, and in accordance with the results in Barro and Sala-i-Martin (2004), the share μ_t^* will become more important than the one for physical and new technology capitals when t goes to infinity. But they will converge to strictly positive values when time goes to infinity.

■

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